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Systematics study of the Z_P model for the ^{235}U + n fission

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Abstract. The parameters of Z_P model for ^{235}U+n fission were derived based upon measured data, with which most of the measured data could be reproduced within their errors, and the unmeasured yields as well as their errors were calculated. The results were improved compared to the others from TALYS calculation and the data libraries such as ENDF-B7-FPY, JEF-3.0/FPY and JENDL-3.3/FPY.

1 Introduction

The independent yields have important applications in the nuclear engineering. The main well known evaluated nuclear data libraries, such as ENDF-B7, JEF-3.0 and JENDL-3.3 have contained the yields of many fission systems [1]. These data libraries were released recently, however some data wherein may actually have not been updated for a quite long time. The motivation of this work is to update these yields based upon measured data with Z_p model.

Currently, the TALYS [2] should be the best program performing theoretical calculation for fission yields, however it had 10–50% of uncertainty, and our preliminary calculation with TALYS did not produce satisfied result. On the other hand, the measured data were inadequate. Therefore, systematics empirical Z_P model, developed by A. Wahl [3,4] and widely used, was adopted in the present work.

According to this model, the fractional independent fission yield FI of a product nuclide with mass number A and charge number Z, could be described with a modified Gaussian distribution,

$$FI(A,Z) = \frac{F(A)N(A)}{\sqrt{\pi}} \int_{Z-1/2}^{Z+1/2} \exp\left[-\left(\frac{Z-Z_P(A)}{\sqrt{2}\,\sigma(A')}\right)^2\right] dZ, \quad (1)$$

$$Z_P(A) = A'\frac{Z_F}{A_F} + \Delta Z_P(A'), \quad (2)$$

$$A_F = A'_L + A'_H, \quad (3)$$

where Z_P and σ are the peak and the width of the Gaussian distribution respectively, while $A' = A + \nu_P(A)$, is the mass of the product fragment right after scission and before the post neutrons emitted to form the product nuclide with mass number A. A' is expressed with A'_L or A'_H for the low and high masses products resp. $\nu_P(A)$ is the average number of the prompt emission neutrons. Z_F is the charge of the virtual compound nucleus with mass A_F. N_A is the normalization factor to make $\sum_Z FI(A,Z) = 1$. The odd-even factor $F(A)$ is described with neutron factor $F_N(A')$ and proton factor $F_Z(A')$ as

$$F(A) = \begin{cases} F_Z(A')\cdot F_N(A'), & (Z = \text{even},\ N = \text{even}),\\ F_Z(A')/F_N(A'), & (Z = \text{even},\ N = \text{odd}),\\ F_N(A')/F_Z(A'), & (Z = \text{odd},\ N = \text{even}),\\ 1/[F_Z(A')\cdot F_N(A')], & (Z = \text{odd},\ N = \text{odd}).\end{cases} \quad (4)$$

The independent yields $Y(A,Z)$ of a chain A can be calculated with equation (5) in the first step assuming $\delta Y_{\rm ch} = 0$, and adjusted to comply with equations (5) and (6) in the second step,

$$Y(A,Z) = FI(A,Z)\cdot \sum_Z Y(A,Z) = FI(A,Z)\cdot (Y_{\rm ch}(A) + \delta Y_{\rm ch}), \quad (5)$$

$$\delta Y_{\rm ch} = \sum_Z \left[Y(A+1,Z)r(A+1,Z) - Y(A,Z)r(A,Z)\right], \quad (6)$$

where $\delta Y_{\rm ch}$ is a small correction between the chain yield $Y_{\rm ch}$ and the summation of the independent fission yields on chain A, due to some independent yields transferred *to* or *from* the other chains with branch r through delayed-neutron decay.

2 Data bases and data process

As no new measured data were found in the recent publishes, the related experimental data, including yields and yield ratios, were retrieved from EXFOR(2006) data library [5]. The chain yields $Y_{\rm ch}(A)$ adopted here were evaluated by T. Liu et al. (2003) [6]. The branches of delayed-neutron decay were taken form refs. [1,7–9].

The yield ratios were converted to absolute yields. Some measured data seemed unreasonable and were not used, for example, which are larger than the chain yield or deviate the Z_P distribution obviously. Systematic errors were assumed to be 3% for those not given in their papers.

Based upon the measured data, the Z_P parameters, ΔZ_P, σ, F_Z and F_N were derived, while the parameter $\nu_P(A)$, as well as the A', Z_F, A_F were adopted from CYF program since they were well done by Wahl [4].

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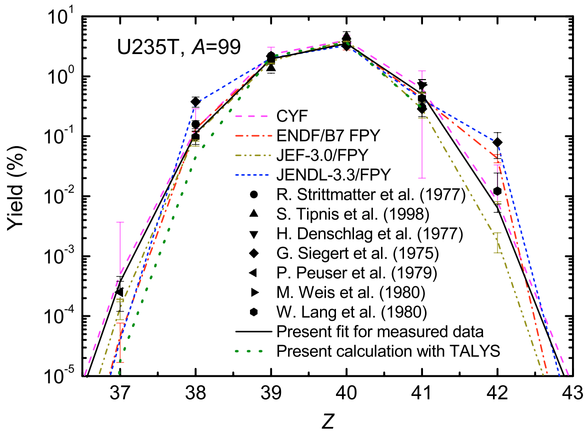

Fig. 1. Fit for measured independent fission yields with Z_P model and yield comparison for U235T on chain $A = 99$. Our fit result and the reproduced yields (systematics calculation, not present in this figure) are well consistent with the measured data. The data on $Z = 38$ and 42 show that the evaluated data from ENDF/B7 FPY and JEF-3.3/FPY need to be updated with the newer measured data from ref. [16].

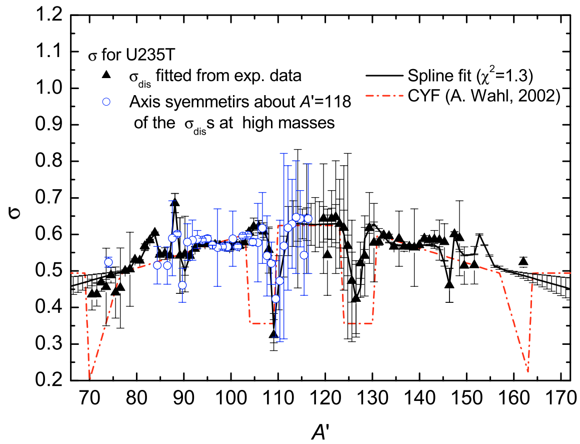

Fig. 2. Systematics parameter $\sigma(A')$ for U235T. The spline fit resulted with reduced $\chi^2 = 1.3$, indicating the fit was quite well. However, the fitted $\sigma(A')$ at center is more uncertain, due to the lack of measured data.

Measured yields on a same chain A were fit with equations (1–4) with least square method to determine the parameters $\Delta Z_P(A)_{\text{dis}}$, $\sigma(A)_{\text{dis}}$, $F_Z(A)_{\text{dis}}$ and $F_N(A)_{\text{dis}}$, here called discrete parameters to distinguish with the systematics parameters $\Delta Z_P(A')$, $\sigma(A')$, $F_Z(A')$ and $F_N(A')$. The systematics parameters as functions of A' were fit with 2 or 3 order of spline functions from the above discrete parameters respectively, and then could be applied to calculate the un-measured yields with equations (1–6). The errors of the systematics parameters as well as the errors of the calculated yields were determined according to the law of the error propagation. Note that the systematic and statistic errors were processed separately.

3 Results and discussion

This paper presents the study of the fissions of ^{235}U induced by thermo-, fission- and high-energy- (~14.8 MeV) neutrons, labelled with U235T, U235F and U235H respectively.

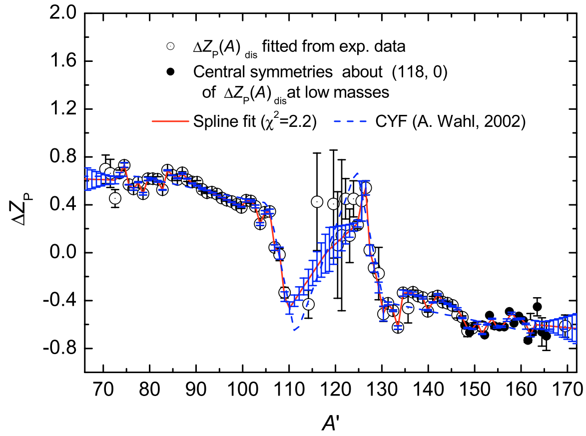

Fig. 3. Systematics parameter $\Delta Z_P(A')$ for U235T. The spline fit resulted with reduced $\chi^2 = 2.2$, indicating the fit was acceptable. However, the fitted $\Delta Z_P(A')$ at center is more uncertain, due to the same reason for the $\sigma(A')$, lack of measured data.

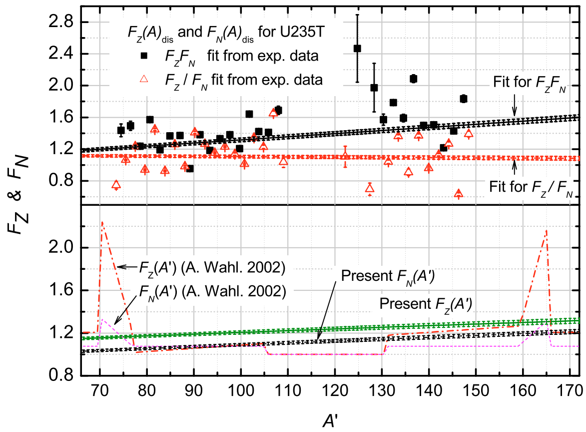

Fig. 4. Parameters F_Z and F_N for U235T. The upper panel is the linear fittings for the discrete F_Z/F_N and $F_Z \cdot F_N$, and the lower one is for the deduced parameters $F_Z(A')$ and $F_N(A')$.

3.1 Result of U235T

84 references concerning U235T were found in the EXFOR2006 (e.g., refs. [10]–[16]), and the measured data wherein covered over 73 chains of A = 70–107, 112, 114, 116, 118–128, 130–148, 150 and 160. The discrete parameters and their errors were obtained based upon these measured data in the way mentioned above. Figure 1 was an instance of the fit of measured data, it showed on chain $A = 99$, the present fit yields were consistent with the measured data except those from Siegert et al. (1975) [13], which were too high compared to the others. The systematics parameters $\Delta Z_P(A')$, $\sigma(A')$, $F_Z(A')$ and $F_N(A')$ and their errors, were reached by spline fitting the discrete parameters and resulted with reduced χ^2s about 4, 1, 70 and 70, respectively, indicating that the fit for the ΔZ_P is acceptable, for the σ is quite well, but for the F_Z and F_N are not satisfied, as shown in figures 2–4. Compared to the systematics parameters from the CYF program, both have similar trends but ours have more fine structures.

The independent yields as well as their errors were calculated with the systematics parameters, and compared to those

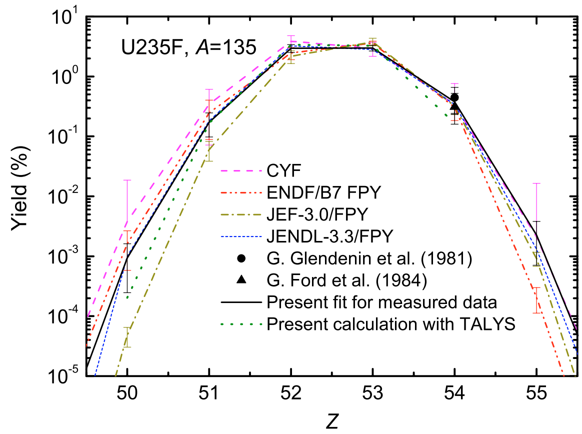

Fig. 5. The independent fission yields for U235F, $A = 135$. The fit was made assuming the primary parameters $\Delta Z_P(A')$, $\sigma(A')$ and $F(A')$ were equal to the data of U235T resp., and then adjust the ΔZ_P ($A = 135$) to fit the measured data.

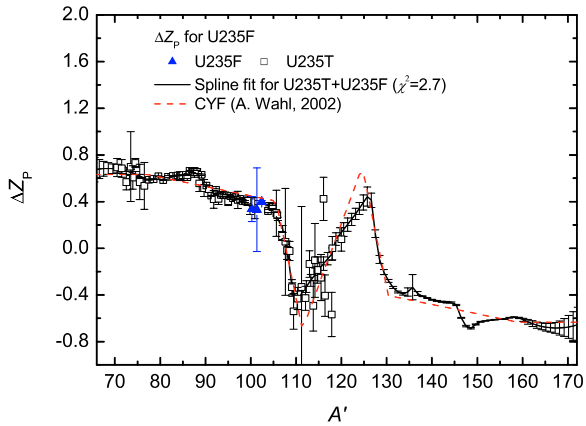

Fig. 6. Systematics parameter ΔZ_P for U235F. Because only three chains had measured data and the consequently $\Delta Z_{P,\text{dis}}$s, the other $\Delta Z_{P,\text{dis}}$s and the systematics parameter $\sigma(A')$ were 'borrowed' from U235T.

from TALYS calculation, ENDF/B7-FPY, JEF-3.0/FPY and JENDL-3.3/FPY. Due to JENDL-3.3/FPY having no errors, it was assumed that JENDL-3.3/FPY have the equal relative errors of ENDF/B7-FPY. The comparisons showed that most of them agree well within the errors, and a few had discrepancies. The following were three examples for chains $A = 85$, 99 and 144. Of the chain $A = 85$, the yields at $Z = 31$–37 agree well, except at $Z = 36$ the yields of JEF-3.0/FPY and JENDL-3.3/FPY were not adjusted to the measured data, and at $Z = 37$, the yield from ENDF/B7-FPY was about 3 orders of magnitude larger than those of the others with unknown reason. Of the chain $A = 99$ (fig. 1), the present systematics yields are well consistent with the measured data except those at $Z = 38$ and $Z = 42$ from ref. [13] (1975), which were too obsolete. While the yield in ENDF/B7-FPY and JEF-3.0/FPY at $Z = 42$ should be updated with the newer data from ref. [16] (Lang et al., 1980). Of the chain $A = 144$, the yields were in agreement well. The result of TALYS calculation for chain $A = 99$ was in agreement with measured data around the peak point ($Z = 39$–41), but not at the left side (fig. 1).

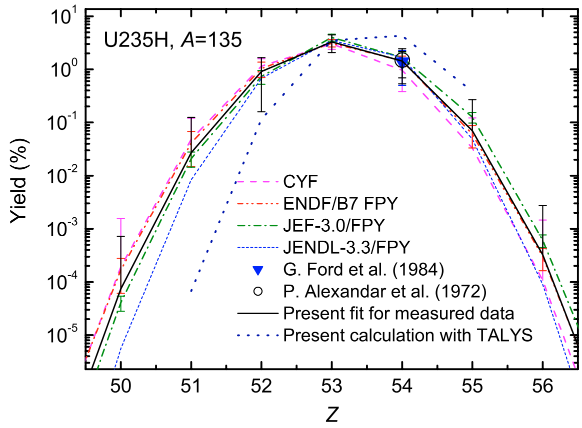

Fig. 7. The independent fission yields for U235H, $A = 135$. This fit was made given the $F(A') = 1$ and $\sigma(A') = 0.62$ by the CYF program.

3.2 Result of U235F

There were only three references [17–19] concerning U235F in EXFOR2006 covering over chains $A = 133$, 134, and 135. These measured data were inadequate to deduce the Z_P model parameters, therefore, the systematics parameters of $F(A')$ (including F_Z and F_N) and $\sigma(A')$ were assumed to equal to those of U235T respectively , and then the discrete $\Delta Z_P(A)_{\text{dis}}$s for chains $A = 133$, 134 and 135 were obtained by fitting these measured data, respectively. For instance, the fit of the chain $A = 135$ was illustrated in figure 5. The systematics parameter $\Delta Z_P(A')$ were reached by spline fitting the above discrete parameters of the three chains combined with those of U235T of the other chains, as shown in figure 6.

The independent yields and errors were calculated with the systematics parameters, but the errors were enlarged by an assumed factor of 1.5 since the parameters $F(A')$ and $\sigma(A')$, and most of the discrete $\Delta Z_P(A)_{\text{dis}}$ were taken from U235T. The results were compared with the others and show that the yields of those nuclides near the peak of the Z_P distribution, or near the measured data were mostly consistent with the others, but the yields at the other points not (e.g., fig. 5), which should be caused by the inadequacy of measured data. The yield calculated with TALYS at $Z = 54$ ($A = 135$) was lower than the measured datum, the yields at other points agreed with others except that from JENDL-3.3/FPY.

3.3 Result of U235H

The situation of U235H is similar to that of U235F, although there were more measured data covering over 18 chains of $A = 82$, 87–93, 96, 121, 123, 133, 135, and 138–142 (e.g., refs. [20]–[21]), they were still inadequate to deduce all the model parameters. Assuming that the the systematics parameters $F(A') = 1$ and $\sigma(A') = 0.620$ given by the CYF program [4], and their uncertainties equal to those of U235T, the discrete parameters ΔZ_Ps were obtained by fitting the measured data, such as the fit of $A = 135$ (fig. 7). Then the systematics parameter $\Delta Z_P(A')$ was deduced by spline fitting the discrete parameters as shown in figure 8.

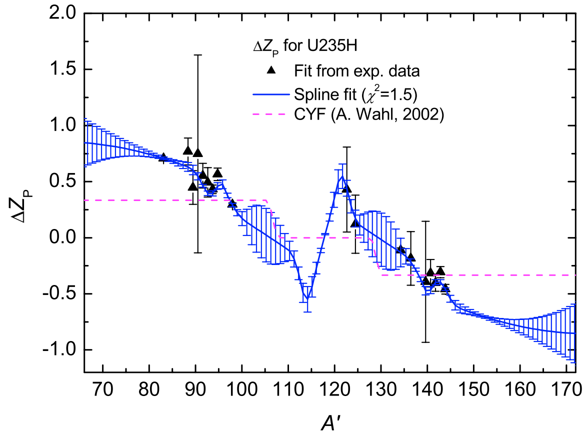

Fig. 8. Systematics parameter ΔZ_P for U235H. Because there are few measured data at the two sides, the parameter where are more uncertain.

The independent yields and errors were calculated and the errors were processed in the same way of U235F, enlarged by a factor of 1.5. The results were compared with the others. As shown in figure 7 for instance, the conclusion of the comparisons is similar to that of U235F, around the peak of Z_P or measured data they agreed well, but had discrepancies at the other points, which was also caused by the inadequacy of measured data. The TALYS calculation for chain $A = 135$ did not give a right peak of the distribution, needed a further study.

4 Summary

The Z_P model was studied to evaluate the independent fission yields for ^{235}U induced by thermal, fission, and about 14 MeV energy neutrons. The parameters of Z_P model were deduced based upon measured data, and were used to calculate the yields and their errors. The yields were compared with the others from TALYS calculations, ENDF/B7-FPY, JEFF-3.0/FPY and JENL-3.3/FPY, the result showed that the present yields as well as their errors were improved.

Compared to U235F and U235H, the measured data for U235T were adequate in some way, the deduced systematics parameters, as well as the calculated independent fission yields were more reliable, and most yields were well in agreement with the data from other data libraries. While for U235F and U235H, due to the lack of measured data, the results had larger uncertainties and discrepancies with others. Hence, more measurements and advanced theories are needed.

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