Sensitivity analysis of critical experiments with evaluated nuclear data libraries

D. Fujiwara and S. Kosaka

Abstract. Criticality benchmark testing was performed with evaluated nuclear data libraries for thermal, low-enriched uranium fuel rod applications. C/E values for $k_{eff}$ were calculated with the continuous-energy Monte Carlo code MVP2 and its libraries generated from ENDF/B-VI.8, ENDF/B-VII.0, JENDL-3.3 and JEFF-3.1. Subsequently, the observed $k_{eff}$ discrepancies between libraries were decomposed to specify the source of difference in the nuclear data libraries using sensitivity analysis technique. The obtained sensitivity profiles are also utilized to estimate the representativity of cold critical experiments to the boiling water reactor under hot operating condition.

1 Introduction

The evaluated nuclear data library ENDF/B-VII.0 was released in December 2006. From the view point of the thermal, low-enriched uranium fuel rod benchmark calculations, the main revised data from ENDF/B-VI.8 library are as follows [1]: the $^{238}$U capture cross section in thermal and resonance range, $^{16}$O(n,$\alpha$) cross section in fast region and $^1$H thermal scattering kernel.

This work is intended to compare the accuracy of criticality prediction with evaluated nuclear data libraries ENDF/B-VI.8, ENDF/B-VII.0, JENDL-3.3, and JEFF-3.1 for Boiling Water Reactor (BWR) applications. The criticality benchmark testing was performed using the existing critical experiments conducted at Toshiba’s Nuclear Critical Assembly (NCA) facility. The discrepancy of the $k_{eff}$ values between libraries were decomposed with sensitivity analyses to understand the physical mechanism caused by the difference of nuclear data. Finally, the representativity of the critical experiments to the application system of BWR in hot operating condition is assessed with sensitivity analysis to confirm the validity of this benchmark testing.

2 Critical experiments

Series of critical experiments at NCA were applied for the criticality benchmark testing. NCA is a tank-type light water moderated criticality facility and its criticality is adjusted by water level. Figure 1 shows the geometrical configuration of NCA core loading, which were arranged to simulate BWR 9x9-type fuel. The core consists of inner 2 x 2 assemblies test region and outer driver region containing 2 wt% enriched UO$_2$ fuel pins. All the experiments were performed at room temperature with full density light water. Reduced water density under hot operating condition was simulated with insertion of aluminum tubes in the test region. The diameter of the aluminum tubes were adjusted for 0% and 40% void ratio. Table 1 shows the arrangement of the test region in each experiment. The geometrical arrangements of the fuel pin and the aluminium tube are illustrated in figure 2.

3 Benchmark calculation

The C/E values for $k_{eff}$ have been calculated for critical experiments using continuous energy Monte Carlo code MVP2 and its libraries generated from ENDF/B-VI.8, ENDF/B-VII.0, JENDL-3.3 and JEFF-3.1. The statistical errors for MVP2 calculation were smaller than 15 pcm. Figures 3, 4, and 5 show the C/E values for each core configuration as a function of hydrogen to heavy metal atomic number ratio (H/HM).

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Table 1. Fuel pin and aluminium tube arrangements of test region for each experiment.

<table>
<thead>
<tr>
<th>Fuel pin arrangement</th>
<th>Al-tube arrangement</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 wt% uniform enrichment</td>
<td>H/HM (void ratio)</td>
</tr>
<tr>
<td>core</td>
<td>6.9 (0%)</td>
</tr>
<tr>
<td></td>
<td>7.2 (0%, 40%)</td>
</tr>
<tr>
<td></td>
<td>7.4 (40%)</td>
</tr>
<tr>
<td>1 wt% and 3wt% enrichment</td>
<td>H/HM (void ratio)</td>
</tr>
<tr>
<td>mismatch core</td>
<td>6.9 (0%)</td>
</tr>
<tr>
<td></td>
<td>7.2 (0%, 40%)</td>
</tr>
<tr>
<td></td>
<td>7.4 (40%)</td>
</tr>
<tr>
<td>2 wt% uniform enrichment</td>
<td>H/HM (void ratio)</td>
</tr>
<tr>
<td>core with gadolinia pin</td>
<td>6.9 (0%)</td>
</tr>
<tr>
<td></td>
<td>7.4 (40%)</td>
</tr>
</tbody>
</table>

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Deviations of the C/E value from unity are 600 ± 100 pcm, 400 ± 100 pcm, 200 ± 100 pcm and 0 ± 100 pcm for ENDF/B-VI.8, JENDL-3.3, JEFF-3.1 and ENDF/B-VII.0, respectively. The agreement of the C/E value to unity is excellent for ENDF/B-VII.0, and the long-standing problem of systematic eigenvalue under-prediction in thermal low-enriched uranium fuel rod systems is improved. However, the dependency of the C/E on the H/HM is still observed for ENDF/B-VII.0. The increment of the C/E value from 0% to 40% void ratio is about ±50 pcm. The existence of the gadolina pin slightly increases the C/E values a few ten pcm for ENDF/B-VII.0, respectively. Note that the capture reactivity contribution to the pin cell $k_{eff}$ change of each nuclide and reaction type is evaluated by equation (1) in which $S$ represents sensitivity coefficient for nuclide $n$, for each reaction type $x$ and for each group $g$. 

$$
(\delta k / k)_x^n = \sum_g S_{xg}^{n,lib2} \sigma_{xg}^{n,lib1} - \sigma_{xg}^{n,lib1} ,
$$

(1)

where $\sigma_{xg}^{n,lib1} = \text{micro cross-section in library "lib1"}$.

Figures 6, 7, and 8 show the reactivity contribution to $k_{eff}$ change from ENDF/B-VI.8 results. Note that the capture reaction in the following figures represents the absorption reaction excluding the fission reaction.

It is clearly observed that the improved $k_{eff}$ C/E values for ENDF/B-VII.0 and JEFF-3.1 are mainly due to a revision in $^{238}$U capture cross section. The contributions of this revision to the increased $k_{eff}$ are about +0.3% $\delta k/k$. The difference in $^{16}$O capture cross section also increase ENDF/B-VII.0 and JENDL-3.3 $k_{eff}$ compared to ENDF/B-VI.8 about +0.1 and +0.2% $\delta k/k$ respectively.

The difference of $^{238}$U elastic scattering cross-section and $\nu$-value between JEFF-3.1 and ENDF/B-VI.8 have contribution of about −0.1% $\delta k/k$ and +0.1% $\delta k/k$ respectively, which
almost cancel out each other. The difference of $^{16}$O elastic scattering cross-section between JENDL-3.3 and ENDF/B-VI.8 has contribution of about $+0.1\% \delta k/k$.

Figures 9 and 10 show energy dependency of the sensitivity and contribution to $k_{eff}$ change from ENDF/B-VI.8 to ENDF/B-VII.0 for $^{238}$U capture and $^{16}$O capture cross-section. The contribution of $^{238}$U capture $+0.3\% \delta k/k$ is caused mainly in thermal and resolved resonance energy range. The magnitude of the contribution due to the thermal range ($<1.0$ eV) is $+0.17\% \delta k/k$ and resolved resonance range (1.0 eV – 20 keV) is $+0.09\% \delta k/k$. These contributions come from the reduced $^{238}$U capture cross-section based on the WPEC sub-group-22 recommendation; smaller thermal capture value 2.683 b at 2200 m/s and smaller effective resonance integral [3]. The contribution of $^{16}$O is due to the smaller $(n,\alpha)$ cross-section in the energy range above 3 MeV.

5 Assessment of representativity

The criticality benchmark testing was performed with critical experiments, in which the moderator densities were set to simulate the BWR hot operating condition. However, they were cold critical experiments under the isothermal room temperature. For discussion of criticality prediction accuracy on the BWR core in hot operating condition, it is important to confirm the representativity of the critical experiments to the applied system. In this study, the representativity is assessed for the 2 wt% enriched pin cell systems under isothermal room temperature and hot operating thermal condition. The latter is conducted with 784 K for fuel and clad temperature and 559 K for moderator temperature.

The representativity is assessed using sensitivity analysis technique, which compare the physical similarity between two systems. A sensitivity-based integral representativity index
Table 2. $E_{sum}$ value for the 2 wt% enriched pin cell systems between isothermal room temperature and hot operating thermal condition.

<table>
<thead>
<tr>
<th>Void ratio</th>
<th>$E_{sum}$ value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>0.930</td>
</tr>
<tr>
<td>40%</td>
<td>0.955</td>
</tr>
</tbody>
</table>

$E_{sum}$ is defined by equation 2, in which $S_{n,g}$ represents sensitivity for each nuclide $n$, for each reaction type $x$, for each group $g$, for the application $a$ and the experiment $e$ [4].

$$E_{sum} = \frac{\sum \sum \sum (S_{n,g}^{e,a} - S_{n,g}^{a,e})}{\left( \sum \sum \sum S_{n,g}^{e,a} \sum \sum \sum S_{n,g}^{a,e} \right)^{1/2}}$$

In this definition, the sensitivity data is thought of as a vector, and the integral index $E_{sum}$ is the cosine of the angle between the two sensitivity vectors for the analyzed systems. Then, the $E_{sum}$ value of 1.0 indicates the parallel sensitivity vector that represents the complete similarity between two systems. As with the case of an $E_{sum}$ value of 0.0 indicates orthogonal vector that represents no similarity. The guideline is recommended that an $E_{sum}$ value of 0.8 or higher indicates enough similarity to the application to be useful in its criticality code validation [5].

The $E_{sum}$ values for the pin cell systems between isothermal room temperature and hot operating thermal condition are evaluated for 0% and 40% void ratio in table 2. Both of the $E_{sum}$ values exceed 0.9 which represents enough similarity regardless of the different thermal condition. These results confirmed the effectiveness of the cold critical experiments for the library validation as against the BWR hot operating condition.

6 Conclusion

The criticality benchmark testing was performed with evaluated nuclear data libraries ENDF/B-VI.8, ENDF/B-VII.0, JENDL-3.3, and JEFF-3.1 for BWR applications. Deviations of $k_{eff}$ $C/E$ value below unity are $600 \pm 100$ pcm, $400 \pm 100$ pcm, $200 \pm 100$ pcm and $0 \pm 100$ pcm for ENDF/B-VI.8, JENDL-3.3, JEFF-3.1 and ENDF/B-VII.0, respectively. The improvement of criticality prediction is excellent for ENDF/B-VII.0, and the long-standing problem of systematic eigenvalue under-prediction in thermal low-enriched uranium fuel rod systems is improved. The dependency of $k_{eff}$ $C/E$ on the H/HM is still observed for ENDF/B-VII.0. The increment of $C/E$ value from 0% to 40% void ratio is about +50 pcm. The existence of gadolinia pin slightly increases the $C/E$ values a few ten pcm for ENDF/B-VI.8, JENDL-3.3 and JEFF-3.1, but no difference was observed for ENDF/B-VII.0.

The sensitivity analyses of pin cell system reveal that the primal contribution to increase in $k_{eff}$ for ENDF/B-VII.0 and JEFF-3.1 compared to ENDF/B-VI.8 is due to the revision in $^{238}$U capture cross-section. The magnitude of the contribution due to the thermal range is about +0.17% $\delta k/k$ and resolved resonance range is about +0.09% $\delta k/k$. The difference in $^{16}$O(n,\alpha) cross-section above 3 MeV also increase ENDF/B-VII.0 and JENDL-3.3 $k_{eff}$ compared to ENDF/B-VI.8 by about +0.1 and +0.2% $\delta k/k$, respectively.

The representativity of cold critical experiments to the BWR hot operating condition is assessed with pin cell calculation using the sensitivity-based integral index $E_{sum}$. The evaluated $E_{sum}$ values exceed 0.9 which represents the effectiveness of the cold critical experiments for the library validation as against the BWR hot operating condition.

References