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Neutron-proton pairing effect on the proton-rich nuclei moment of inertia

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Abstract. The neutron-proton (n-p) pairing effect on the nuclear moment of inertia is studied within the BCS approximation in the isovector case. An analytical expression of the moment of inertia is established by means of the cranking model. This expression generalizes the usual BCS one (i.e. when only the pairing between like-particles is considered). The moment of inertia of $N = Z$ even-even nuclei, for which experimental values are known, i.e., such as $32 \leq A \leq 80$, has been numerically evaluated, with and without inclusion of the n-p pairing effect. The used single-particle and eigen-states are those of a deformed Woods-Saxon mean field. It turns out that the inclusion of the n-p pairing improves the obtained values when compared to the usual BCS approximation, since the average discrepancies with the experimental data are respectively 7% and 37%.

1 Introduction

Pairing correlations between nucleons are an important part of the description of nuclear behavior. In heavy nuclei, such correlations are usually neglected on the grounds that the two Fremi levels are far apart. In nuclei with $N \simeq Z$ however, the Fermi levels are close and neutron-proton (n-p) pairing correlations can be expected to play a significant role in nuclear structure. Effects associated with proton-proton (p-p) and neutron-neutron (n-n) pairs are well understood, but n-p pairing is a phenomenon which has only recently been opened up for experimental investigation [1–6]. On the other hand, the moment of inertia is a very important physical quantity that plays an essential role in the description of the nuclear rotation motion. Since it is sensitive to the deformation, its measurement can provide indications about the nuclear shape [7]. But, for "ordinary" nuclei, it is well known that there exists a discrepancy of the order of 10–40% between the theoretical BCS values and the experimental ones [7]. In order to reduce this discrepancy several works have been performed [8–14]. However, these works deal with "ordinary" nuclei and do not include the n-p pairing correlations. The latter have been taken into account by Gerceklioglu et al. [15,16] in the calculation of the moment of inertia of several rare-earth and actinide nuclei. They consider a special type of n-p pairing interaction which is assumed to be a weak residual force. However, their calculations deal with nuclei such that the smallest value of the neutron excess (N-Z) is 26. But it is well known that the n-p pairing correlations rapidly disappear when (N-Z) increases [17,18]. Other studies deal with the moment of inertia of nuclei such as $N = Z$ [3,6] but they don't give explicitly the moment of inertia expression.

The purpose of the present work is to study the influence of the n-p isovector pairing effect, within the BCS approximation framework, on the moment of inertia of the proton-rich even-even nuclei for which $N = Z$, and of whose experimental values of the moment of inertia are known. In this aim an analytical expression of the moment of inertia that includes the n-p pairing correlations is established in the next section. The numerical results are presented and discussed in section 3.

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2 Formalism

2.1 Hamiltonian

In the second quantization and isotopic spin formalism, the Hamiltonian of a system constituted by N neutrons and Z protons, when we restrict ourselves to the isovector case, is given by

$$H = \sum_{\substack{\nu>0 \\ t}} \varepsilon_{\nu t} \left(a^\dagger_{\nu t} a_{\nu t} + a^\dagger_{\tilde{\nu} t} a_{\tilde{\nu} t} \right) - \frac{1}{2} \sum_{tt'} G_{tt'} \sum_{\nu\mu>0} \left(a^\dagger_{\nu t} a^\dagger_{\tilde{\nu} t'} a_{\tilde{\mu} t'} a_{\mu t} + a^\dagger_{\nu t} a^\dagger_{\tilde{\nu} t'} a_{\tilde{\mu} t} a_{\mu t'} \right) \quad (1)$$

where the t subscript corresponds to the isotopic spin component ($t = \mathrm{n, p}$), and $a^\dagger_{\nu t}$ and $a_{\nu t}$ respectively represent the creation and annihilation operators of the particle in the state $|\nu t\rangle$, of energy $\varepsilon_{\nu t}$, $|\tilde{\nu} t\rangle$ is the time reverse of $|\nu t\rangle$ and $G_{tt'}$ characterizes the pairing-strength. The neutrons and protons are supposed to occupy the same energy levels. In the BCS approach, one introduces two Lagrange parameters λ_t ($t = \mathrm{n, p}$) that allow one to define the auxiliary Hamiltonian

$$H' = H - \sum_t \lambda_t N_t \quad (2)$$

with $N_t = \sum_{\nu>0} \left(a^\dagger_{\nu t} a_{\nu t} + a^\dagger_{\tilde{\nu} t} a_{\tilde{\nu} t} \right) \quad t = \mathrm{n, p}.$

H is then approximately diagonalized using the Bogoliubov-Valatin transformation [19]

$$\alpha^\dagger_{\nu\tau} = \sum_t \left(u_{\nu\tau t} a^\dagger_{\nu t} + v_{\nu\tau t} a_{\tilde{\nu} t} \right) \qquad \tau = 1, 2 \quad (3)$$

where $\alpha^\dagger_{\nu\tau}$ is the quasi-particle creation operator and τ refers to the quasi-particle type.

Using the transformation given by equation (3), H becomes

$$H = E_0 + H_{11} + H_{res} \tag{4}$$

where E_0 is the ground state energy given by

$$\begin{aligned} E_0 = {} & 2\sum_{\substack{\nu>0 \\ t}} \varepsilon_{\nu t}\left(\sum_{\tau} v_{\nu\tau t}^2\right) \\ & -\frac{1}{2}\sum_{\substack{tt' \\ \nu>0}} G_{tt'}\left[\left(\sum_{\tau} v_{\nu\tau t}v_{\nu\tau t'}\right)^2 + \left(\sum_{\tau} v_{\nu\tau t}^2\right)\left(\sum_{\tau} v_{\nu\tau t'}^2\right)\right] \\ & -\frac{1}{2}\sum_{tt'} G_{tt'}\sum_{\nu,\mu>0}\left[\left(\sum_{\tau} v_{\nu\tau t}u_{\nu\tau t'}\right)\left(\sum_{\tau} u_{\mu\tau t}v_{\mu\tau t'}\right)\right]. \end{aligned} \tag{5}$$

H_{11} is the independent quasi-particle Hamiltonian but in a non-diagonal form

$$H_{11} = \sum_{\substack{\nu>0 \\ \tau\tau'}} E_{\nu\tau\tau'}\left(\alpha_{\nu\tau}^{\dagger}\alpha_{\nu\tau'} + \alpha_{\tilde{\nu}\tau}^{\dagger}\alpha_{\tilde{\nu}\tau'}\right), \tag{6}$$

$E_{\nu\tau\tau'}$ being the quasi-particle energy given by

$$\begin{aligned} E_{\nu\tau\tau'} = {} & \sum_{t}\left[\varepsilon_{\nu t} - \frac{1}{2}\sum_{t'} G_{tt'}\sum_{\tau} v_{\nu\tau t'}^2\right](u_{\nu\tau t}u_{\nu\tau' t} - v_{\nu\tau t}v_{\nu\tau' t}) \\ & -\frac{1}{2}\sum_{tt'} G_{tt'}\left[\left(\sum_{\tau} v_{\nu\tau t}v_{\nu\tau t'}\right)(u_{\nu\tau t}u_{\nu\tau' t'} - v_{\nu\tau t}v_{\nu\tau' t'})\right] \\ & -\sum_{tt'} \varDelta_{tt'}\left(u_{\nu\tau t}v_{\nu\tau' t'} + u_{\nu\tau' t}v_{\nu\tau t'}\right) \end{aligned} \tag{7}$$

with

$$\varDelta_{tt'} = -G_{tt'}\sum_{\substack{\nu>0 \\ \tau}} (u_{\nu\tau t}v_{\nu\tau t'} + u_{\nu\tau t'}v_{\nu\tau t}) \tag{8}$$

and H_{res} represents the residual term which is neglected in the independent quasi-particle approximation.

As underlined above, H_{11} is not diagonal, hence one has now to perform a new diagonalization that leads to [20]

$$H_{11} = \sum_{\nu\tau} \lambda_{\tau}^{\nu}\beta_{\nu\tau}^{\dagger}\beta_{\nu\tau} \tag{9}$$

with

$$\beta_{\nu\tau}^{\dagger} = \sum_{k} x_{\tau k}^{\nu}\alpha_{\nu k}^{\dagger} \tag{10}$$

where λ_{τ}^{ν} and $x_{\tau k}^{\nu}$ are given by

$$\lambda_{1,2}^{\nu} = \frac{1}{2}\left[(E_{\nu 11} + E_{\nu 22}) \pm \sqrt{(E_{\nu 11} - E_{\nu 22})^2 + 4E_{12}^2}\right] \tag{11}$$

and

$$\begin{aligned} x_{11}^{\nu} &= \frac{E_{\nu 12}}{\sqrt{E_{\nu 12}^2 + \left(\lambda_1^{\nu} - E_{\nu 11}\right)}}, \quad x_{21}^{\nu} = \frac{\lambda_2^{\nu} - E_{\nu 22}}{\sqrt{E_{\nu 12}^2 + \left(\lambda_2^{\nu} - E_{\nu 22}\right)}} \\ x_{12}^{\nu} &= \frac{\lambda_1^{\nu} - E_{\nu 11}}{\sqrt{E_{\nu 12}^2 + \left(\lambda_1^{\nu} - E_{\nu 11}\right)}}, \quad x_{22}^{\nu} = \frac{E_{\nu 12}}{\sqrt{E_{\nu 12}^2 + \left(\lambda_2^{\nu} - E_{\nu 22}\right)}}. \end{aligned} \tag{12}$$

In fact, this process corresponds to the definition of new quasi-particles, of energies λ_{τ}^{ν}, created by the $\beta_{\nu\tau}^{\dagger}$ operators and hence to a generalized Bogoliubov-Valatin transformation given by

$$\beta_{\nu\tau}^{\dagger} = \sum_{t}\left(U_{\nu\tau t}a_{\nu t}^{\dagger} + V_{\nu\tau t}a_{\tilde{\nu}t}\right) \tag{13}$$

with

$$U_{\nu\tau t} = \sum_{j=1}^{2} x_{\tau j}^{\nu}u_{\nu jt}, \quad V_{\nu\tau t} = \sum_{j=1}^{2} x_{\tau j}^{\nu}v_{\nu jt} \tag{14}$$

where $\tau = 1, 2$ and $t =$ p, n.

2.2 Moment of inertia

The ground state of an even-even nucleus is given by [20]

$$|\varPsi\rangle = \prod_{\nu>0,\tau} |\varPsi_{\nu}\rangle \tag{15}$$

where

$$\begin{aligned} |\varPsi_{\nu}\rangle = {} & \{B_1^{\nu}a_{\tilde{\nu}p}^{\dagger}a_{\nu p}^{\dagger}a_{\tilde{\nu}n}^{\dagger}a_{\nu n}^{\dagger} + B_p^{\nu}a_{\tilde{\nu}p}^{\dagger}a_{\nu p}^{\dagger} + B_n^{\nu}a_{\tilde{\nu}n}^{\dagger}a_{\nu n}^{\dagger} \\ & + B_4^{\nu}\left(a_{\tilde{\nu}p}^{\dagger}a_{\nu n}^{\dagger} + a_{\tilde{\nu}n}^{\dagger}a_{\nu p}^{\dagger}\right) + B_5^{\nu}\}|0\rangle \end{aligned} \tag{16}$$

where the B_i^{ν} depends upon U and V parameters. The excited states for a system of which one particle is blocked in the state $|kT\rangle$ and another in the state $\left|\tilde{l}T'\right\rangle$ where $k \neq l$ and $T, T' = n, p$ are given by

$$\left|kT\tilde{l}T'\right\rangle = K_{kl}^{TT'}a_{kT}^{\dagger}a_{\tilde{l}T'}^{\dagger}\prod_{\substack{j>0 \\ j\neq k\neq l}} \left|\varPsi_j\right\rangle \tag{17}$$

where $K_{kl}^{TT'}$ is a normalization constant. In the quasi-particle representation, they become

$$\left|kT\tilde{l}T'\right\rangle = K_{kl}^{TT'}\sum_{\tau\tau'} \gamma_{k\tau T}\gamma_{l\tau' T'}\beta_{k\tau}^{\dagger}\beta_{\tilde{l}\tau'}^{\dagger}|\varPsi\rangle \tag{18}$$

with

$$K_{kl}^{TT'} = \left(\sum_{\tau\tau'} \gamma_{k\tau T}^2\gamma_{l\tau' T'}^2\right)^{-\frac{1}{2}}$$

$$\text{and} \quad \gamma_{\nu\tau T} = B_5^{\nu}U_{\nu\tau T} + B_T^{\nu}V_{\nu\tau T} + B_4^{\nu}V_{\nu\tau T'}.$$

Their energies reads

$$\begin{aligned} E_{kl}^{TT'} = {} & E_0 + \left(K_{kl}^{TT'}\right)^2 \\ & \left[\left(\sum_{\tau} \lambda_{\tau}^{k}\gamma_{k\tau T}^2\right)\left(\sum_{\tau} \gamma_{l\tau T'}^2\right) + \left(\sum_{\tau} \lambda_{\tau}^{l}\gamma_{l\tau T'}^2\right)\left(\sum_{\tau} \gamma_{k\tau T}^2\right)\right]. \end{aligned} \tag{19}$$

The moment of inertia of a rotating system calculated within the Inglis cranking method is given by [21]

$$\Im_C = 2\hbar^2\sum_{\substack{\mu \\ \mu\neq 0}} \frac{|\langle\mu|J_x|0\rangle|^2}{E_{\mu} - E_0} \tag{20}$$

where E_μ is the energy of the excited state $|\mu\rangle$ and E_0 that of the ground state $|0\rangle$.

In the framework of the present study, the ground state is the state (15) of energy E_0. The excited states that contribute to the moment of inertia are those of the form (18) of energy $E^{TT'}_{\nu\mu}$. A straightforward calculation leads to the following expression of the moment of inertia including the n-p pairing effect

$$\Im_{np} = 2\hbar^2 \sum_{\substack{\nu,\mu \\ \nu\neq\mu, TT'}} \frac{\left| \sum_{tt'\tau\tau'} K^{TT'}_{\nu\mu} \gamma_{\nu\tau T}\gamma_{\mu\tau' T'} \langle \nu t | j_x | \mu t' \rangle W_{\nu\mu} \right|^2}{E^{TT'}_{\nu\mu} - E_0} \qquad (21)$$

with $W_{\nu\mu} = \left(U_{\mu\tau' t'} V_{\nu\tau t} - U_{\nu\tau t} V_{\mu\tau' t'} \right)$.

Let us notice that equation (21) is more deeply modified by the inclusion of the n-p pairing effect than that established by Gerceklioglu et al., in ref. [15,16], within a study based on a special type of n-p pairing interaction which is assumed to be a weak residual force. The latter is given by

$$\Im_{n(p)} = 2\hbar^2 \sum_s \frac{|\langle s | J_x | s' \rangle|^2}{\tilde{\varepsilon}_s + \tilde{\varepsilon}_{s'}} (\tilde{u}_s \tilde{v}_{s'} - \tilde{u}_{s'} \tilde{v}_s)^2 \qquad (22)$$

for the neutron (respectively proton) system, with

$$\tilde{u}_s = \left[\frac{1}{2} \left\{ 1 + \frac{(E_s - \lambda_{n(p)})}{\tilde{\varepsilon}_s} \right\} \right]^{\frac{1}{2}},$$

$$\tilde{v}_s = \left[\frac{1}{2} \left\{ 1 - \frac{(E_s - \lambda_{n(p)})}{\tilde{\varepsilon}_s} \right\} \right]^{\frac{1}{2}}, \qquad (23)$$

E_s being the single particle energies and $\tilde{\varepsilon}_s$ is defined by

$$\tilde{\varepsilon}_s = \sqrt{\left(E_s - \lambda_{n(p)} \right)^2 + \left(\varDelta_{n(p)} + \frac{q}{2} \varDelta_{p(n)} \right)^2}. \qquad (24)$$

Indeed equation (22) is formally the same as the usual BCS one [7]. In fact, in this expression, only the gap parameters are modified by the inclusion of the n-p pairing effect, i.e., $\varDelta_{n,(p)} = \varDelta_{n,(p)} + \frac{1}{2} q \varDelta_{p,(n)}$ where q is an adjustable parameter.

3 Numerical results and discussions

We consider nuclei such as $N = Z$ and $16 \le Z \le 40$ (for which experimental values of the moment of inertia are known). In these nuclei, the experimental values of the three pairing gap parameters are known and have the same values [5,22]

$$\varDelta^{\exp}_{pp} = \varDelta^{\exp}_{nn} = \varDelta^{\exp}_{np}.$$

The pairing-strength constants G_{pp}, G_{nn}, and G_{np} are then determined to reproduce the latter, that are deduced from the odd-even mass differences [23].

The single-particle energies and states used in the present work are those of the Woods-Saxon mean field. The ground state deformation parameters have been extracted from ref. [23].

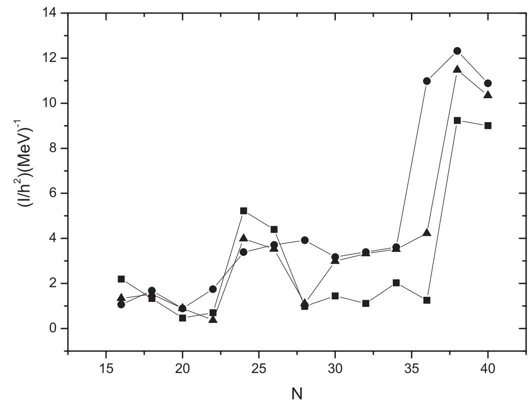

Fig. 1. Variation of the moment of inertia as a function of the neutron number N, for nuclei such as $N = Z$, and of whose experimental values are known; without (■) and with (●) inclusion of the n-p pairing effect. (▲) represent the experimental values.

We have evaluated the moment of inertia $(\Im/\hbar^2)(MeV)^{-1}$ of these nuclei in their ground state, with and without inclusion of the n-p pairing. The corresponding results are reported in figure 1 as a function of the neutron number N. We have also reported the experimental values. The latter are deduced from the 2^+ state of the experimental spectrum of ref. [24], except that of the ${}^{72}_{36}$Kr and ${}^{68}_{34}$Se nuclei that come from ref. [25] and that of the ${}^{76}_{38}$Sr nucleus that come from ref. [26]. From figure 1 one can notice that:

- The contribution of the n-p pairing, when compared to that of the pairing between like-particles is important. Indeed, the mean relative discrepancy defined as $\frac{|\Im_{np} - \Im_{BCS}|}{\Im_{np}}$ is 30%.
- The inclusion of the n-p pairing considerably improves the moment of inertia values, when compared to the pairing between like-particles ones. Indeed, the mean relative discrepancy, defined as $\frac{|\Im_{th} - \Im_{\exp}|}{\Im_{\exp}}$ where $\Im_{\exp}$ is the experimental value and $\Im_{th}$ the theoretical prediction, is 7% and 37% respectively, when the extreme points are excluded with and without inclusion of the n-p pairing. It is thus necessary to take into account the n-p pairing effect for the study of the moment of inertia for this kind of nuclei.

4 Conclusion

In the present work, we have studied the isovector n-p pairing effect on the nuclear moment of inertia.

It has been noticed that the usual Bogoliubov-Valatin transformation does not lead to an independent quasi-particle Hamiltonian. A generalized Bogoliubov-Valatin transformation has thus been defined. The latter leads to a renormalization of the quasi-particle energies. The ground state and the excited ones, as well as the corresponding energies have been established.

Afterwards, an analytical expression of the moment of inertia that explicitly depends upon the n-p pairing has been established within the Inglis cranking model.

The model has been applied for nuclei such as $N = Z$ and whose experimental values of the moment of inertia are known (i.e., such as $16 \leq Z \leq 40$). The inclusion of the n-p pairing correlations reduces the mean relative discrepancy between the experimental and theoretical values from 30% to 7%. The inclusion of the isovector pairing effects is thus necessary for the evaluation of the moment of inertia of such nuclei.

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