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Fluctuations of fission data in the resonances with a channel-mode formalism

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Abstract. This work investigates the modeling of the fluctuations of fission observables (such as the neutrons and γ multiplicity, the average kinetic energy of fragments) in the resonance range. A formalism combining fission channels and fission modes gives a tractable methodology to study this longstanding issue. The present work explores the channel-mode formalism but includes also the effect of prefission γ. The case of ^{235}U fission below 20 eV is studied.

1 Introduction

The fluctuations of fission data such as neutron prompt multiplicity $\langle\nu\rangle$[1] in resonances of actinides may have a significant impact on integral parameters in reactor calculations. Because of contradictory experimental data and a lack of understanding of the underlying mechanisms, these fluctuations are not represented in the recent evaluated nuclear-data files such as JEFF3.1 or ENDF/B-VII.

Experiments performed by Hambsch et al. [1] on ^{235}U demonstrated the correlations between the fluctuations of total kinetic energy (TKE) of fragments and the fluctuations of the fragment mass distributions. In the framework of fission mode formalism such as the Multi-Modal Random Neck Rupture (MM-RNR) model proposed by Brosa et al. [2], the TKE fluctuations were explained by the fluctuations in the weight of fission modes (standard-I , standard-II, and superlong).

A formalism combining the fission channels and fission modes was presented by Furman [4] in order to relate the observed fluctuations to the transition-state characteristics (such as spin, parity and spin projection onto the symmetry axis of the fissioning nucleus: $J^\pi K$). The present work is an attempt to use this formalism to describe the fluctuations of $\langle\nu\rangle$ and other characteristics of ^{235}U below 20 eV.

2 Channel-mode formalism

Low-energy nuclear fission has been historically interpreted within two different pictures: resonance analysis mostly relies on the concept of fission channels associated with transition states, describing a fissioning nucleus during its deformation to scission [5], while the properties of fission fragments is best viewed as a multimodal process. To connect the two frameworks and establish a more general channel-mode formalism, Barabanov and Furman [3], [4] described the fragmentation as a binary nuclear reaction. With the help of the helicity concept, they defined a scattering matrix connecting transition-state wave functions and fission-fragment exit channels. This formalism was able to predict the fragments angular distribution with polarized neutrons and investigate parity-violating effect in fission. After summation over fission-fragment characteristics, Furman obtained a multimode scattering matrix and justified the use of a natural extension of the fission cross section $\sigma_f^{J^\pi K,m}$ which includes channel quantum numbers $J^\pi K$ and mode m.

$$\sigma_f(E) = \sum_{J^\pi} \sigma_f^{J^\pi}(E) = \sum_{J^\pi K} \sigma_f^{J^\pi K}(E) = \sum_{J^\pi K m} \sigma_f^{J^\pi K,m}(E). \quad (1)$$

E is the incident neutron energy. Defining:

- $P^J(E) = \sigma_f^J(E)/\sigma_f(E)$, $P^{JK}(E) = \sigma_f^{JK}(E)/\sigma_f^J(E_n)$;
- and $w_m^{JK} = \sigma_f^{J^\pi K,m}(E)/\sigma_f^{JK}(E)$.

In the channel-mode formalism, the yield $Y(E, M, TKE)$ of fission-fragment of mass M and total kinetic energy TKE becomes:

$$Y(E, M, TKE) = \sum_m Y_m(M, TKE) \sum_{J^\pi K} w_m^{JK} P^J(E) P^{JK}(E) \quad (2)$$

where $Y_m(M, TKE)$ is the fission yield curve for the mode m as defined in multimodal models, and is independent of the incident neutron energy. The second sum represents the energy-dependent weight of the mode m. If we assume that the main origin of the fluctuations of direct fission data [excluding the (n,γf) process] in the resonances is the fluctuations of fission-fragment yields, the same formalism can be used for post-scission observables. In that case, any fission characteristics averaged over fission fragments, such as $\langle\nu\rangle$, $\langle E_\gamma\rangle$ or $\langle TKE\rangle$ can be put in the following form:

$$\langle X\rangle(E) = \sum_m \langle X_m\rangle \sum_{JK} w_m^{JK} P^J(E) P^{JK}(E). \quad (3)$$

The parameters related to channels such as $P^J(E)$ and $P^{JK}(E)$ are strongly fluctuating with incident neutron energy

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[1] The bracket ⟨⟩ means an average over fission fragments.

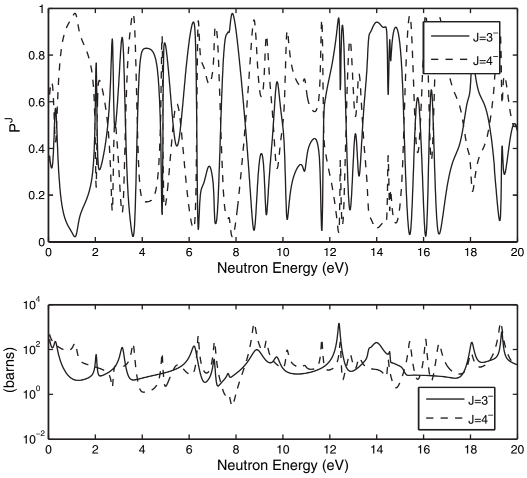

Fig. 1. The upper figure shows the fission probabilities $P^J(E)$ and the lower figure displays the spin-separated fission cross sections for J = 3^- and J = 4^-. Data were calculated by Kopach and co-workers [6].

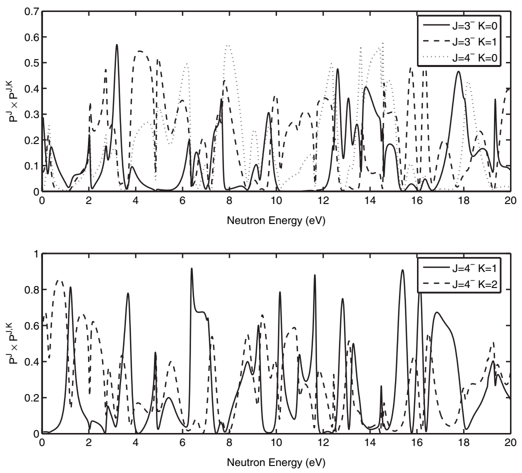

Fig. 2. Fission probabilities $P^J \times P^{JK}(E)$ for J = 3^- (upper figure) and J = 4^- (lower figure) from Kopach and co-workers [6].

while the mode parameters $\langle X_m \rangle$ are constants (see figs. 1 and 2). The present work assumes that the probabilities w_m^{JK} for a given fission channel $J^\pi K$ to feed the mode m do not depend on incident neutron energy.

3 Effect of the (n,γf) reaction

As discussed by many authors, the (n,γf) reaction (emission of γ-rays before fission) may compete with direct fission. Because pre-fission γ emission decreases the average fragment excitation energy, this process can contribute significantly to the fluctuations of the fission data in the resonances. The correlations between measured reciprocal fission widths and release γ-energy in resonance provided a reliable estimate of the effect. However, the properties of transition states (quantum numbers, fission barriers) of the residual fissioning nucleus after γ emission are not well known and the application of the channel-mode formalism is not possible. Therefore, in this work, direct and (n,γf) reactions were treated with a different formalism. Experimental data $\langle \nu \rangle$, $\langle TKE \rangle$ and $\langle E_\gamma \rangle$ measured in various energy ranges $\Delta E_{\lambda'}$ were corrected to subtract the (n,γf) contribution according the phenomenological formula:

$$\langle X \rangle_{corr}^{\lambda'} = \langle X \rangle_{exp}^{\lambda'} - \frac{1}{\Delta E_{\lambda'}} \int_{\Delta E_{\lambda'}} \sum_J \frac{\sigma_{n\gamma f}^J}{\sigma_f^J} \langle E_{\gamma f}^J \rangle \left(\frac{\partial \langle X \rangle}{\partial E^*} \right)^J dE. \quad (4)$$

$\langle E_{\gamma f}^J \rangle$ is the average energy released by prefission γ and $\left(\frac{\partial \langle X \rangle}{\partial E^*}\right)^J$ is the derivative of the fission characteristics $\langle X \rangle$ with respect to compound-nucleus excitation energy.

4 Application to ^{235}U fission

The formalism was applied to the fission of ^{235}U in the resonance range. Three fission channels are open at low incident neutron energy for J = 3^- resonances (K = 0, 1, 2) and two for J = 4^- (K = 1, 2).

- Only Standard-I and Standard-II modes are considered. The symmetric superlong mode is neglected.
- The (n,γf) pointwise cross section was calculated with a Single Level Breit Wigner Formalism, using an average radiation width of $\Gamma_{\gamma,f}$ = 4.7 meV for J = 3^- and $\Gamma_{\gamma,f}$ = 2.1 meV for J = 4^-. From the work of Trochon [7], we adopted $\langle E_{\gamma f}^J \rangle$ = 0.85 MeV for J = 3^- and $\langle E_{\gamma f}^J \rangle$ = 0.80 for J = 4^-.
- $P^J(E)$ and $P^{JK}(E)$ were taken from the resonance analysis performed by Kopach et al. [6] below 20 eV.
- To determine the channel-mode weight w_m^{JK}, we use the multimodal analysis of Hambsch of experimental fragment-mass and TKE distributions. In several energy ranges λ, Hambsch analyzed his data to deduce the weigths of the mode $w_{\lambda,m}^{exp}$. In the present work, the w_m^{JK} were fitted using

$$w_{\lambda,m}^{exp} = \sum_{JK} w_m^{JK} \frac{1}{\Delta E_\lambda} \int_{\Delta E_\lambda} P^J(E) P^{JK}(E) dE. \quad (5)$$

  Unfortunately, the experimental values of $w_{\lambda,m}^{exp}$ given by Hambsch neglects the (n,γf) reaction. It introduced here an inconsistency in the application of the formalism that cannot be tackled easily.
- Using the fitted w_m^{JK}, the average modal prompt multiplicity $\langle \nu_m \rangle$ were assessed using measured data of Gwin et al. [8] and Howe et al. [9] below 20 eV. The Gwin data have been renormalized to the known thermal value ν_{th} = 2.420 n/fission and the Howe data have been renormalized to the average value measured by Gwin in the 0.5 – 10 eV energy range. (n,γf) corrections were made using equation (4) using $(\partial \langle \nu \rangle / \partial E^*)^J$ = 0.131 for both spins, from the work of Schackleton [10]. Original experimental data were published over energy ranges $\Delta E_{\lambda'}$ which are different from the measurements of Hambsch.

$$\langle \nu_{\lambda'}^{exp} \rangle = \sum_m \langle \nu_m \rangle \sum_{JK} w_m^{JK} \frac{1}{\Delta E_{\lambda'}} \int_{\Delta E_{\lambda'}} P^J(E) P^{JK}(E)\, dE. \quad (6)$$

Table 1. Main parameters of the model for ^{235}U.

Mode Parameters		
Variable	Standard-I	Standard-II
$\langle \nu_m \rangle$	1.937 ± 0.2	2.537 ± 0.05
$\langle TKE_m \rangle$ (MeV)	179.6 ± 1.6	168.6 ± 0.4
$w_m^{J=3^-,K=0}$	0.2220	0.7780
$w_m^{J=3^-,K=1}$	0.2533	0.7457
$w_m^{J=3^-,K=2}$	0.1450	0.8550
$w_m^{J=4^-,K=1}$	0.1974	0.8026
$w_m^{J=4^-,K=2}$	0.1912	0.8088
Channel Parameters		
$P^J(E)$, $P^{JK}(E)$	Taken from Kopach et al. [6]	
(n,γf) Correction Parameters		
$\Gamma_{\gamma,f}^{J=3}$ - $\langle E_{\gamma f}^{J=3} \rangle$	4.7 MeV – 0.85 MeV	
$\Gamma_{\gamma,f}^{J=4}$ - $\langle E_{\gamma f}^{J=4} \rangle$	2.1 MeV – 0.80 MeV	
$(\partial\langle \nu \rangle/\partial E^*)^{J=3,4}$	0.1310	
$(\partial\langle TKE \rangle/\partial E^*)^{J=3,4}$	−0.8874	

Similarly, experimental data of $\langle TKE \rangle$ of Hambsch over the same energy ranges as $w_{\lambda,m}^{exp}$ permitted the determination of $\langle TKE_m \rangle$ for standard-I and standard-II mode. Experimental $\langle TKE \rangle$ were corrected for prefission γ using equation (4) with $(\partial\langle TKE \rangle/\partial E^*) = -0.8874$ extrapolated from the experimental values of $TKE/(TKE + TXE)^2$ at higher energy measured by Hambsch et al. [11].

- Eventually, pointwise values of $\langle \nu \rangle$, $\langle TKE \rangle$ and $\langle E_\gamma \rangle$ versus neutron energy E were obtained with w_m^{JK}, $\langle \nu_m \rangle$, $\langle TKE_m \rangle$, $\langle E_{\gamma m} \rangle$, $P^J(E)$ and $P^{JK}(E)$ using equation (3).

Table 1 summarizes the main parameters of the models determined in the present work. We note a high sensitivity of w_m^{JK} to the quantum number K for J$^\pi$ = 3$^-$.

5 Results

A way to test the consistency of the model is to compare the mode weights calculated with equations 5 with the fitted w_m^{JK} and experimental data $w_{\lambda,m}^{exp}$. Figure 3 shows a fair agreement but the observed discrepancies may have a significant effect on the final values of the observables.

Average mode parameters are in good agreement with the previous work of Gohs [12] who used a different method. The comparison between calculated $\langle \nu \rangle$ and experimental data are shown in figure 4. We note that the (n, γf) effect is small but not negligible. The structures of the measured data is quite well reproduced in the range 3–12 eV with the present calculation. Nevertheless, our calculations do not explain the large fluctuations measured by Gwin between 0.3 and 1 eV. Discrepancies are observed and likely come from several sources: the inaccurate determination of w_m^{JK} (due to uncertainties in the Hambsch measurements of fission yields and TKE) or errors in $P^J(E)$ and $P^{JK}(E)$ determination or uncertainties on Howe and Gwin experimental data.

[2] TXE is Total Excitation Energy of fission fragment.

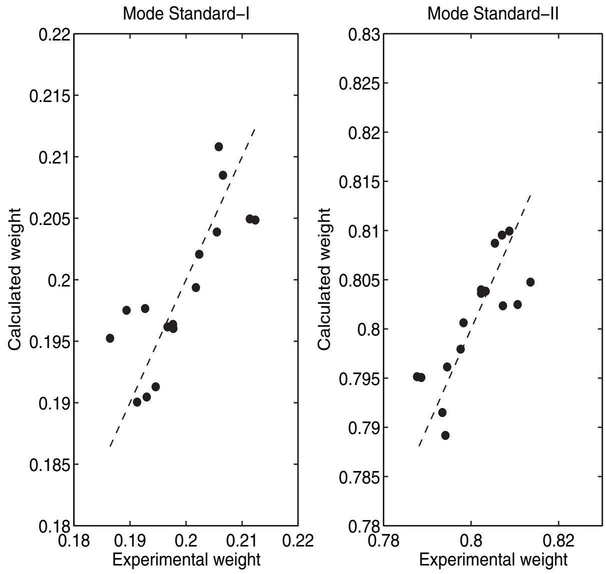

Fig. 3. Calculated mode weights $w_{\lambda,m}^{calc}$ (averaged in several energy ranges ΔE_λ) with fitted w_m^{JK} compared with experimental values $w_{\lambda,m}^{exp}$.

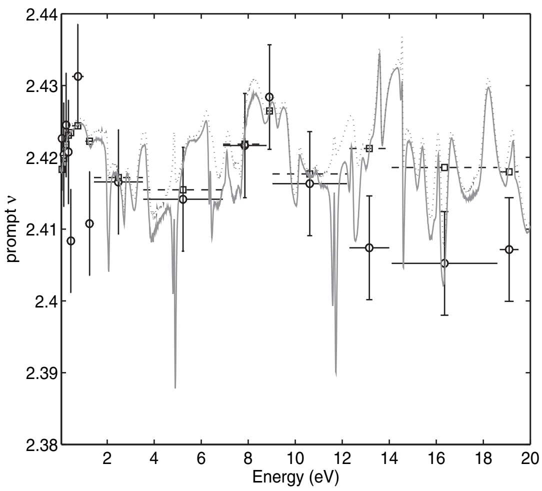

Fig. 4. Comparison between measured neutron multiplicity $\langle \nu \rangle$ and calculations with the present formalism (pointwise and averaged over the same energy ranges as in the measurements). The white dots are experimental data and the white squares are the calculated values. Straight line is the pointwise calculated multiplicity and dot line is the same without (n,γf) corrections.

Figure 6 shows the comparison between calculated TKE and experimental data of Hambsch. TKE fluctuations are not well reproduced: low values of TKE are overestimated by the calculations while high values are underestimated. Again, discrepancies could be explained by errors in the determination of the fitted parameters of the model. The present formalism could be tested against γ-energy measurements in the resonances. Unfortunately, publications of existing measurements do not specify the energy-ranges boundaries for each measured resonance. Therefore the comparison is meaningless because the present work predicts large fluctuations of fission data within a single resonance.

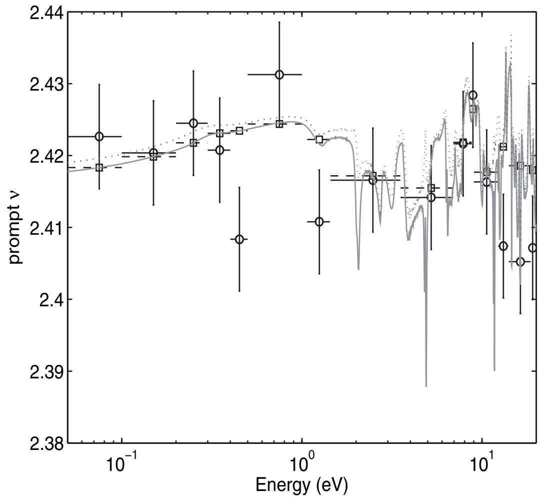

Fig. 5. Same figure as figure 4 in logarithmic scale.

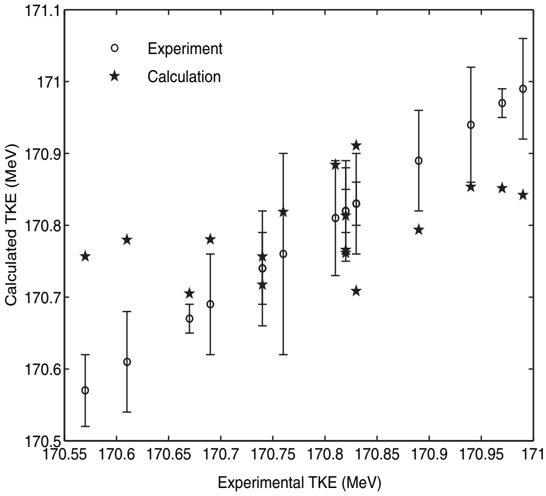

Fig. 6. Comparison between measured total kinetic energy $\langle TKE \rangle$ and calculations with the present formalism (pointwise and averaged over the same energy ranges as in the measurements).

6 Conclusion

The formalism used in this work seems promising. It parameterizes the complexity of the fission process with a relatively limited number of parameters related to both fission-channel and fission-mode characteristics. It permits to predict and extrapolate some fission data where measurements are not available and could be a useful tool for evaluation. This formalism could be improved by including the conservation of total energy in fission. Nevertheless, the determination of the parameters of the model (spin J- and K-separated cross section, weights of the mode, average mode values) requires accurate and specific measurements which are still not available today. The application of this formalism to ^{235}U stresses the need for further measurements, namely:

- The measurement of $\langle \nu \rangle$ in the resonance range were performed more than 30 years ago. High-resolution measurements with modern experimental techniques would be valuable.
- The separation of fission cross section in J and K componant is crucial in the application of this formalism. Recent resonance-parameter sets [6] and [13] provide discrepant decompositions which need to be clarified. A proper account of (n,γf) reaction should also be included in the resonance analysis.
- Simultaneous measurements of fission-fragment yields versus mass, and total kinetic energy $Y(E, M, TKE)$, as well as neutron and γ multiplicities would be of interest for ^{235}U but also for ^{239}Pu.

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