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Surrogate reactions: the Weisskopf-Ewing approximation and its limitations

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Abstract. A brief description of the Surrogate reaction method, an indirect approach for determining compound-nuclear reaction cross sections, is presented. The Weisskopf-Ewing limit, an approximation scheme that is typically employed in the analysis of Surrogate experiments, is considered and its validity and limitations are discussed.

1 Introduction

Knowledge of nuclear reaction cross sections is essential for a wide range of nuclear physics applications and indirect methods play an important role in determining the cross sections of interest. Often the cross section needed for a particular application cannot be measured directly since the relevant energy region is inaccessible or the target is too short-lived. While most indirect methods currently under consideration focus on direct-reaction cross sections, there is a clear need for compound-nuclear reaction cross sections.

The Surrogate technique aims at determining the cross section for a two-step ("desired") reaction that proceeds through a compound nuclear state by using a combination of experiment and theory. The compound nucleus is produced by means of an alternative ("Surrogate") reaction, and the desired cross section is obtained by combining a calculation of the compound-nucleus formation in the desired reaction with a measurement of the compound-nucleus decay in the Surrogate reaction. The Surrogate approach was first employed in the 1970s [1] to estimate neutron-induced fission cross sections from transfer reactions. More recently, the Surrogate method has attracted renewed attention: additional fission experiments have been carried out [2–4], new approximation schemes [5–8] and refinements [9] of the method have been studied, and applications to other mass regions are being considered [10]. Here we discuss some of the limitations of the current approximations and outline some recommendations for future theory and experimental developments.

1.1 The Surrogate method

Surrogate reaction methods are based on the assumption that the formation and decay of a compound nucleus (CN) are independent of each other (for each angular momentum and parity value). The relevant CN is formed in a "Surrogate" ($d + D \rightarrow b + B^*$) reaction rather than in the "desired" reaction ($a + A \rightarrow B^* \rightarrow c + C$) and the desired reaction cross section is obtained via a combination of experimental observation and modeling, using statistical Hauser-Feshbach theory.

In the Hauser-Feshbach formalism [11], the cross section for this desired reaction takes the form:

$$\sigma_{\alpha\chi}(E_a) = \sum_{J,\pi} \sigma_\alpha^{CN}(E, J, \pi) \;\; G_\chi^{CN}(E, J, \pi), \qquad (1)$$

with α and χ denoting the relevant entrance and exit channels, $a + A$ and $c + C$, respectively. The excitation energy E of the compound nucleus, B^*, is related to the projectile energy E_a via the energy needed for separating a from B: $E_a = E - S_a(B)$. In many cases the formation cross sections $\sigma_\alpha^{CN}(E, J, \pi)$ can be calculated to a reasonable accuracy by using optical potentials, while the theoretical decay probabilities $G_\chi^{CN}(E, J, \pi)$ for the different decay channels are often quite uncertain. The objective of the Surrogate method is to determine or constrain these decay probabilities experimentally. In the Surrogate approach, the compound nucleus B^* is produced by means of an alternative, direct (Surrogate) reaction, $d + D \rightarrow b + B^*$, and the desired decay channel $\chi(B^* \rightarrow c + C)$ is observed in coincidence with the outgoing particle b. The coincidence measurement provides

$$P_{\delta\chi}(E) = \sum_{J,\pi} F_\delta^{CN}(E, J, \pi) \;\; G_\chi^{CN}(E, J, \pi), \qquad (2)$$

the probability that the compound nucleus was formed in the Surrogate reaction with spin-parity distribution $F_\delta^{CN}(E, J, \pi)$ and subsequently decayed into the channel χ. The spin-parity distributions $F_\delta^{CN}(E, J, \pi)$, which may be very different from the compound-nuclear spin-parity populations following the absorption of the projectile a in the desired reaction, have to be determined theoretically, so that the branching ratios $G_\chi^{CN}(E, J, \pi)$ can be extracted from the measurements.

In practice, the decay of the compound nucleus is modeled and the $G_\chi^{CN}(E, J, \pi)$ are obtained by fitting the calculations to reproduce the measured decay probabilities and subsequently inserted in equation (1) to yield the desired cross section [9]. Alternatively, approximations to the full Surrogate formalism outlined here can be employed. Almost all applications to date

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have relied on approximation schemes, such as the Weisskopf-Ewing limit of the Hauser-Feshbach theory or the Surrogate Ratio approach. In the Ratio method, the measurements are carried out on two similar targets, for one of which the desired reaction has been accurately measured. Further details and tests of this method may be found in ref. [5]. The Ratio method assumes the validity of the Weisskopf-Ewing approximation, but may reduce the effects of modest violations of this approximation on the extracted cross sections. Below, we discuss some of the limitations of the Weisskopf-Ewing approximation.

2 Validity of current approximations

Under certain conditions [12,13] the branching ratios that enter in the Hauser-Feshbach formalism become independent of the angular momentum and parity, $J\pi$, of the CN, and the expression for the desired CN cross section becomes

$$\sigma_{\alpha\chi}^{WE}(E) = \sigma_{\alpha}^{CN}(E) \times \mathcal{G}_{\chi}^{CN}(E), \tag{3}$$

where $\sigma_{\alpha}^{CN}(E) = \sum_{J,\pi} \sigma_{\alpha}^{CN}(E, J, \pi)$ is the reaction cross section describing the formation of the CN at energy E via the entrance channel α, and $\mathcal{G}_{\chi}^{CN}(E)$ denotes the $J\pi$-independent branching ratio for the exit channel χ. This is the Weisskopf-Ewing limit [14] of the Hauser-Feshbach theory.

Most applications of the Surrogate method so far have been based on the assumption that the Weisskopf-Ewing limit is valid for the cases of interest. The Weisskopf-Ewing approximation greatly simplifies the application of the method: it becomes straightforward to obtain the $J\pi$-independent branching ratios $\mathcal{G}_{\chi}^{CN}(E)$ from measurements of $P_{\delta\chi}(E)[= \mathcal{G}_{\chi}^{CN}(E)$, since $\sum_{J\pi} F_{\delta}^{CN}(E, J, \pi) = 1]$ and to calculate the desired reaction cross section. Calculating the direct-reaction probabilities $F_{\delta}^{CN}(E, J, \pi)$ and modeling the decay of the compound nucleus are no longer required.

Unfortunately, it is not feasible to determine the branching ratios $G_{\chi}^{CN}(E, J, \pi)$ for individual $J\pi$ values experimentally and thus test under which conditions the Weisskopf-Ewing limit is applicable. The validity of the approximation is typically established by comparing cross sections obtained from Surrogate experiments that were analyzed in the Weisskopf-Ewing limit with independent direct measurements or cross section evaluations. However, it is possible to carry out calculations to predict the behavior of the branching ratios $G_{\chi}^{CN}(E, J, \pi)$ as a function of energy, angular momentum, and spin, and to draw some conclusions about the limitations of the Weisskopf-Ewing approximation. Such calculations have been carried out for the decay of the CN ^{235}U by fission and γ-emission [5]. We will discuss some of the implications of these calculations in sections 2.1 and 2.3 below. The results of analogous calculations, carried out for the decay of near-spherical nuclei in the mass-90 region [15], are considered in section 2.4. Experimental evidence for deviations from the Weisskopf-Ewing limit has been observed in recent experiments that aimed at investigating spin effects in Surrogate reactions [16]. Results from that work are considered in section 2.2.

2.1 Fission probabilities and the WE approximation

While the branching ratios $G_{\chi=fission}^{CN}(E, J, \pi)$ cannot be directly measured in a fission experiment, they can be extracted from a calculation of the (n,f) cross section and their $J\pi$-dependence can be studied. To this end, we simulated a nuclear reaction. We extracted the branching ratios from a full Hauser-Feshbach calculation of the ^{235}U(n,f) reaction that was calibrated to an evaluation of experimental data. The model used a deformed optical potential and the level schemes, level densities, gamma strength functions, fission-model parameters, and pre-equilibrium parameters were adjusted to reproduce the available data on n-induced fission for energies from $E_n = 0$ to 20 MeV. For more details see ref. [5]. In figure 1 we present the extracted $G_{fission}^{CN}(E, J, \pi)$ for fission proceeding through positive parity states in the compound nucleus ^{236}U. The top panel shows the $G_{fission}^{CN}(E, J, \pi)$ for $J = 0, 5, 10, 15, 20$ for neutron energies $E_n = 0 - 20$ MeV ($E_n = E(^{236}U) - S_n(^{236}U)$. We observe that the branching ratios exhibit a significant $J\pi$ dependence, in particular for low neutron energies, $E_n = 0 - 5$ MeV. With increasing energy, the differences decrease, although the discrepancies become more pronounced near the thresholds for second-chance and third-chance fission. The branching ratios for negative parity states (not shown) are very similar.

In the bottom panel of figure 1, a narrower range of angular-momentum values is considered for neutron energies up to 7 MeV. The associated branching ratios are seen to be very similar to each other for all but the lowest energies. The comparison of the top and bottom panels illustrates an important point: it is not a priori clear whether the Weisskopf-Ewing limit applies to a particular reaction in a given energy regime. While the Weisskopf-Ewing approximation may break down for a reaction that populates a wide range of $J\pi$ states, it may provide a valid description for a reaction that populates a narrow range of angular-momentum values. Thus, it becomes important to obtain information on the spin-parity distributions of the decaying CN.

2.2 Impact on extracted fission cross sections

Given the dependence of the branching ratios $G_{\chi}^{CN}(E, J, \pi)$ on the spin of the decaying CN, illustrated above, it is relevant to consider the impact of neglecting this dependence on cross sections extracted from Surrogate experiments. Employing the Weisskopf-Ewing assumption in the analysis of Surrogate reactions for which this approximation is not valid will result in extracted cross sections that deviate from the desired true cross section. This has been shown in ref. [5], where the simulated branching ratios shown in figure 1 were taken to represent the "true" branching ratios and employed to simulate coincidence probabilities $P_{\chi}(E)$ with the help of several schematic CN spin-parity distributions. The calculated $P_{\chi}(E)$ correspond to the coincidence probabilities that are observed in typical Surrogate experiments. These $P_{\chi}(E)$ were then analyzed in analogy to actual experimental results, and the Weisskopf-Ewing approximation was applied in the analysis. The resulting (n,f) cross sections were found to depend on

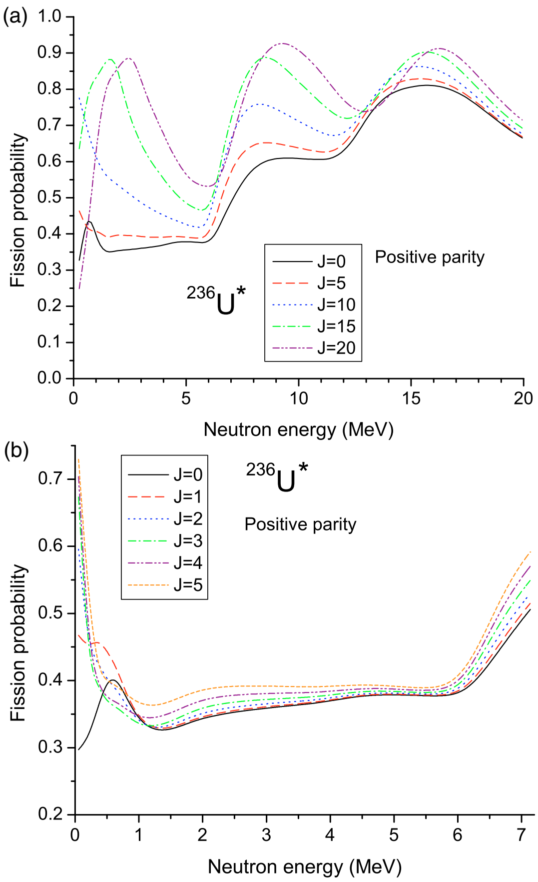

Fig. 1. Calculated branching ratios $G^{CN}_{fission}$ for fission of $^{236}U^*$, as a function of the laboratory neutron energy in the ^{235}U + n system. Results are shown for positive parity states with total angular momenta $J = 0, 5, 10, 15, 20$ (top panel) and $J = 0, 1, 2, 3, 4, 5$ (bottom panel) in the compound nucleus $^{236}U^*$.

the $J\pi$ distribution selected in the analysis: For example, depending on the $J\pi$ distribution selected, the ^{235}U(n,f) cross sections extracted from the simulated experiments differed from the expected results by up to 15%–20% for neutron energies above 5 MeV, and up to 50% for smaller energies (not shown here; for details, see ref. [5]).

Since the angular-momentum transfer between projectile and target in a direct (Surrogate) reaction $d + D \rightarrow b + B^*$ depends on the angle of the outgoing particle b, the measured coincidence probabilities $P_\chi(E)$ should depend on that angle if the Weisskopf-Ewing approximation is not valid. Such angular dependence was indeed observed in a recent experiment carried out at the 88-inch Cyclotron at Lawrence Berkeley National Laboratory: a 42-MeV ^{3}He beam was used to create the CN ^{237}U via the $^{238}U(^3He, \alpha)$ Surrogate reaction [16]. The α-fission coincidence probabilities were measured and the ^{236}U(n,f) cross section was determined, using the Weisskopf-Ewing approximation. Restricting the analysis to α-fission coincidence events for which the outgoing α particle was observed at angles between 36° and 45° relative to the beam axis led to a cross section that is different from the cross section obtained for α particles observed in the 57° to 62° range, for neutron energies below about 1.5 MeV. This is illustrated in figure 2, where the cross sections extracted for the two angular ranges are compared to each other and to the cross section that is obtained by averaging over the full angular range. The findings demonstrate that more theoretical and experimental work is required to fully understand the population and decay of compound nuclear systems and to improve the accuracy and reliability of cross sections extracted from Surrogate experiments, in particular for low energies (below about 2 MeV).

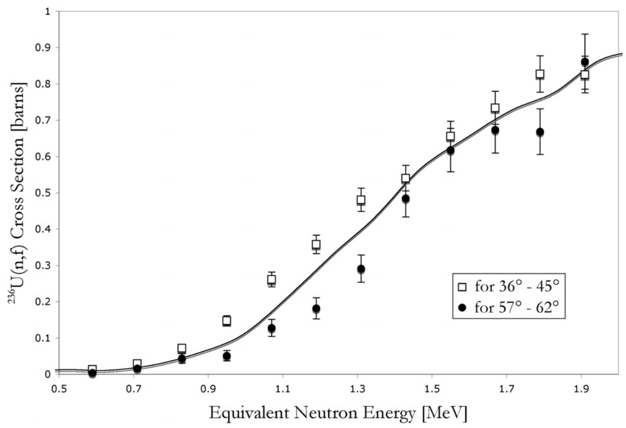

Fig. 2. ^{236}U(n,f) cross section obtained from a Weisskopf-Ewing analysis of Surrogate $^{238}U(^3He,\alpha)$ measurements. Data represented by open squares correspond to events for which the outgoing α particle was observed at 36° to 45°, while filled circles correspond to an angular range of 57° to 62°. The solid line is the cross section that results from averaging over all experimentally accessible angles, 36° to 62°.

2.3 Angular-momentum effects in neutron-capture reactions for actinide nuclei

Surrogate experiments that aim at determining CN (n,γ) cross sections have to detect the outgoing direct-reaction particle b in coincidence with an observable that identifies the γ-emission decay channel. For even-even CN this is typically accomplished by gating on coincidences between the outgoing particle b and individual γ-rays that are characteristic of transitions between low-lying levels of the ground band of the decaying nucleus. (A reaction-model calculation of the γ-cascade is then required to connect the observed gamma yields to the quantity that is needed, the sum of all cascades.)

It is known that γ-ray yields from a decaying compound nucleus depend very sensitively on the spin distribution present in the CN prior to the decay. The effect is illustrated in figure 3 for the decay of the CN ^{236}U, formed in the n+^{235}U and n+^{235m}U channels, respectively. The plot shows the ratio of the calculated intensity of a particular γ-ray to the total number of γ-cascades that eventually reach the ground state of ^{236}U. The simulation described in section 2.1 above was employed to calculate the relevant intensities. Both panels of figure 3 show relative γ-yields for the decay of the CN ^{236}U as a function of energy. Apart from the intensities for the $2^+ \rightarrow 0^+$ transition, the yields shown in the two panels of the figure are very different from each other. This difference can be attributed to a difference in the $J\pi$ distribution in the decaying CN. The CN ^{236}U associated with the upper panel is expected to have a spin distribution that is peaked at higher angular-momentum values than the CN ^{236}U associated with the lower panel. The former CN was produced in a reaction in which a neutron was absorbed by the $J\pi = 7/2^-$ ground state of ^{235}U, while the latter was produced in a reaction involving the first excited state of ^{235}U, which has angular momentum and parity $J\pi = 1/2^+$. The energy difference between these two target states is very small, less than 100 eV, thus the only significant difference between the CN ^{236}U produced in these reactions is the spin-parity population of the decaying nucleus. It is evident that the CN spin distribution has a significant influence on the observed quantities and thus on cross sections that are extracted under the assumption that the Weisskopf-Ewing limit is valid.

While the strong dependence of the γ-ray yields on the $J\pi$ distribution of the CN makes the extraction of a (n, γ) cross section from a Surrogate experiment difficult, this sensitivity also provides an opportunity for obtaining information on the spin-parity distribution of the decaying nucleus from an observation of the associated γ-rays. In fact, γ-rays observed in recent Surrogate measurements provided guidance for selecting the most likely $J\pi$ distribution used in ref. [5]. Measurements of yields for various individual γ rays will provide stringent tests for theoretical predictions of the formation and decay of a compound nucleus produced in a Surrogate reaction.

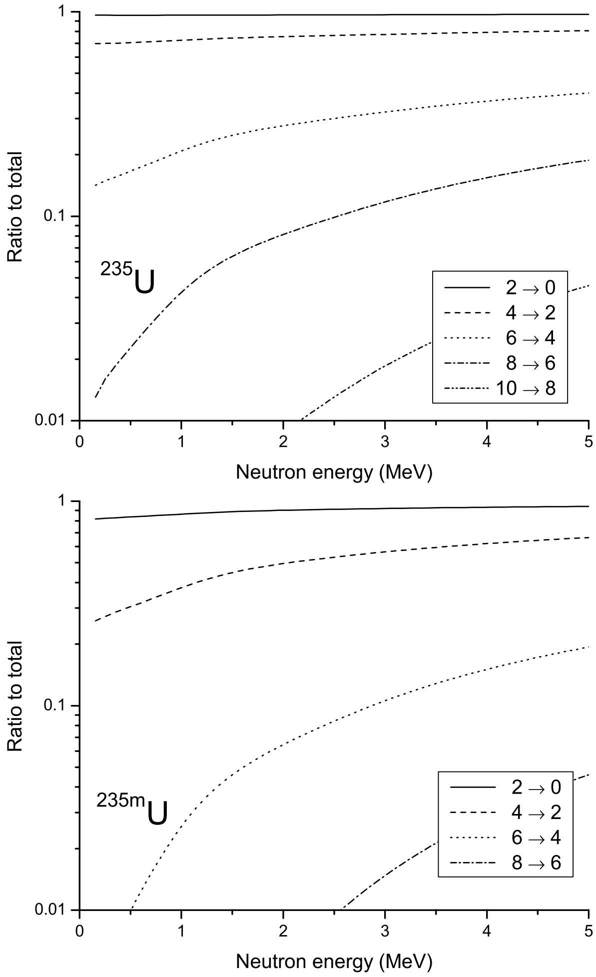

Fig. 3. Ratio of yields of various γ-rays for transitions in the ground-state band of ^{236}U to the total production of ^{236}U. The upper panel shows results for the decay of ^{236}U following its production in the n +^{235} U channel, while the lower panel is for the n +^{235m} U channel.

2.4 Neutron capture reactions for near-spherical nuclei – pushing the limits of the method?

The dependence of the decay probabilities $G_\chi^{CN}(E, J, \pi)$ on the $J\pi$ population of the CN provides a major challenge for Surrogate applications to nuclei near closed shells, as can be inferred from the ^{91}Zr(n,γ) example presented in figure 4. Shown is the probability that a ^{92}Zr state with excitation energy $E = S_n + E_n$ and given $J\pi$ value decays via γ-emission. For neutron energies below about 2.5 MeV, the gamma decay probabilities depend very sensitively on angular momentum and parity. In this energy region, the decay of the ^{92}Zr compound nucleus proceeds exclusively by gamma- or neutron-emission. Due to the low level density in the neighboring ^{91}Zr nucleus, only a very small number of neutron decay channels are open. This circumstance, and the fact that the neutron transmission coefficients are very large for the s and p wave channels, and small for all other channels, leads to gamma-decay probabilities that are very sensitive to the $J\pi$ population of the decaying compound state. It is clear that the Weisskopf-Ewing approximation is not valid in this region. With increasing energy, more levels in ^{91}Zr become available and the dependence of the decay probabilities on angular momentum and parity becomes weaker. The situation is also expected to improve as one moves away from closed-shell nuclei. For example, ^{91}Zr has only one level below 1 MeV (the ground state), while the deformed nucleus ^{155}Gd has over 60. Consequently, the decay probabilities for ^{156}Gd can be expected to depend more smoothly on energy and to exhibit less sensitivity to the $J\pi$ values of the compound nucleus. Surrogate reactions for ^{155}Gd (n, γ) are currently under investigation. The fact that the ^{91}Zr (n, γ) reaction involves a target very near a closed shell makes this reaction a particularly difficult candidate for a Surrogate treatment. For energies below 2.5 MeV, the Weisskopf-Ewing approximation cannot be employed and the accuracy of the cross section obtained from a full Surrogate analysis will be limited, since small errors in the predicted Surrogate $J\pi$ population $F_\delta^{CN}(E, J, \pi)$ introduce large uncertainties in the extracted decay probabilities. However, even in this extreme case, it may be possible to obtain some useful reaction information from a Surrogate experiment. Since Surrogate experiments can provide coincidence probabilities $P_{\delta\chi}(E)$ for a wide range of energies, one can study a region for which the Weisskopf-Ewing limit is approximately valid ($E_n > 2.5$ MeV in the current example) and use the results to normalize the calculated decay probabilities. The deduced normalization factor can subsequently be used in the statistical reaction calculation of the cross section in the desired energy range. This approach has shown some promising results [10].

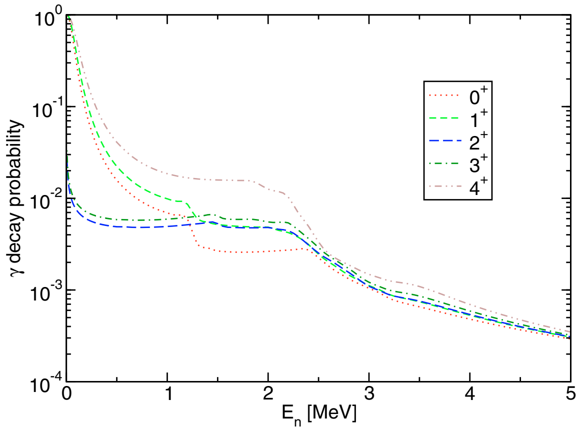

Fig. 4. Gamma branching ratios as a function of spin, parity, and energy in ^{92}Zr. The energy given is the equivalent neutron energy, $E_n = E - S_n$, where E denotes the excitation energy in ^{92}Zr.

3 Summary and outlook

Renewed interest in the surrogate method over the last few years has led to significant improvements in our understanding of the method as well as to a deeper appreciation of what remains to be done. Some of the recent accomplishments, outstanding problems, and recommendations are the following:

- Recent experiments have been largely devoted to fission. The Weisskopf-Ewing method appears to be satisfactory above 1 to 2 MeV, but a full Hauser-Feshbach treatment may be needed at lower energies. This is confirmed by the work of Younes and Britt [9].
- The Ratio method is useful in reducing errors in certain cases when the conditions for validity of the Weisskopf-Ewing approximation are not well satisfied.
- When the Weisskopf-Ewing approximation cannot be used, information on $J\pi$ distributions is crucial. These must be supplied by suitable direct-reaction calculations leading to the excitation of high-lying final states. Developing reliable calculations for the wide variety of applicable direct reactions (stripping, pickup, inelastic scattering, and charge exchange) is a challenge for the theorists.
- Yields of low-lying gammas following gamma cascades de-exciting high-lying compound states can give useful information on $J\pi$ distributions to supplement the results of direct-interaction calculations. This should be exploited experimentally.
- Surrogate measurements of (n, γ) reactions should be carried out in both spherical and deformed nuclei. Such measurements are important for understanding the limits of the surrogate-reaction technique and for determining whether the technique is useful for determining (n,γ) cross sections at low energies relevant for astrophysics.
- The assumption in the surrogate-reaction process that the result of the initial direct reaction is a fully-equilibrated compound nucleus is not obviously correct and needs to be checked by appropriate calculations. For example, a neutron deposited in the target nucleus at sufficiently high excitation energies by a (d,p) reaction may be emitted into the continuum before the compound nucleus is formed.

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